Monte Carlo Methods in Finance Using Fat Tail Models

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June 19, 2012

Abstract

Random regular variation, volatility, and uncertainty are facts of everyday life. We don’t know what the weather or stock market activity will be on any given day, but most days deliver fairly common occurrences and familiar kinds of changes. Extreme and improbable events can occasionally happen as well; rivers flood, hurricanes surge, currencies topple, and markets crash. Financial professionals incorporate all the ups and downs of markets into their expectations for portfolios and financial plans. The problem lies with understanding and planning for normal market variations and also properly including an appropriate probability for rare but very extreme risks. This brief note describes some features of Monte Carlo Simulation, specifically fat tails, which offers an additional method to model market risks.

Monte Carlo Simulation

Stochastics, or Monte Carlo Simulation, is often used by financial planners to refine their illustrations of how different retirement strategies might perform under the dynamic stresses of potential economic futures (Hertz, 1970; Boyle, 1977; Fishman, 1996). These methods expose potential weaknesses in financial plans by including the results from thousands, sometimes millions, of potential future economic outcomes. The resulting set of possible futures contains the full range of market fluctuations. The advantage of Monte Carlo analysis over deterministic methods lay in Monte Carlo’s ability to examine risk and volatility as a fact of financial life.

Fat Tailed Probability Curves

While Monte Carlo techniques present many advantages over deterministic analytical strategies, the method can have shortcomings if distribution assumptions are violated (i.e. data from the actual events being modeled are not Normally distributed). In nature, mathematics, and engineering, a distribution assumption used for simulation, statistical analysis, or hypothesis testing is often the Normal distribution (Kmenta, 1997; Glasserman, 2000). Data, from nature, often fall into the familiar bell-shaped curve (i.e. tree ring growth, rain fall, and student height). The distribution is often sufficient for finance too; it has been used extensively to model financial systems (Black and Scholes, 1973; Laise, 2009; Lindsey, 1996). In the extremities of the distribution however, the Normal distribution can fail under certain situations, like its representation of rare, but extreme events (i.e. stock market crashes, black swans, good news events). For these rare events, fat tail distributions may more accurately represent probability of these phenomenon occurring (Viswanathan et al., 2002; Kaplan 2009).

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Using Fat Tails in Financial Planning

Financial advisors have the option to utilize Fat tail return distributions in their Monte Carlo illustrations with fat tail modeling systems available in Silver Financial Planner and Total Planning Suite from Money Tree Software. Selecting the fat tail option implements a modified return probability array that adds an additional level of stress testing to detailed financial plans that may better reflect actual market risk distribution. This works to ensure advisors can more accurately model for capital sufficiency, portfolio stability, and capture potential strategic opportunities resulting from extreme market events.

In Money Tree's Fat tail probability models, the set of possible future return streams is implemented using a two-stage approach for each year within the 10,000 simulation runs generated to model the risks facing each financial plan. First, normally distributed random returns are generated with a user-defined mean and standard deviation. This mimics a population of normal volatility when returns are considered typical. When one of these first-stage random numbers falls within the fat tail region, the number is randomly regenerated to mimic the more extreme lower fat tail return distribution (i.e. rare, but extreme downturn events).

Throughout the simulation, the resulting 0.14% of resampled second stage values more accurately represents the occurrence of rare and extreme financial events. This fat tail implementation provides a consistent technique that 1) mimics the majority of the most common returns as provided by the normal distribution, and 2) better models the occurrence of very unusual and extreme outlying events not sufficiently represented by the Normal distribution.

A Fat Tail Example

Figure 1: Comparison of results with and without fat tails implementation.

Figure 1 displays results from two identical Monte Carlo simulation runs; the example on the left displays the distribution of the terminal wealth fat tails option turned off, the example on the right is the terminal wealth distribution with the fat tails option turned on.

The main difference between the two result sets is a reduced and somewhat broader range of projected wealth at life expectancy as a result of the fat tail probability distribution. This more conservative collection of results is due to the increased number of very negative returns in the fat tail area in relation to the slightly increased right skewness of positive returns. Overall, the more conservative wealth projections occur due to the compounding effect of more extreme negative returns occurring beyond negative three standard deviations. In the fat tail model, these extreme negative returns are pushed even further to the negative to more accurately mimic actual financial market behavior.
References


